Statistical detection of fraud in the reporting of Croatian public companies

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Abstract

Statistical methods based on Benford's distribution, Z- and χ^2-statistics are being successfully applied to detect likely accounting and reporting fraud, for example in the daily usage of the Internal Revenue Service in the USA, and in historical analysis of Greek macroeconomic reporting. We adapt and apply the methodology to the analysis of the reporting of some leading Croatian public companies. We find indications of reporting fraud in several of the companies analyzed. In particular we find correlation between the likelihood of reporting fraud, measured as a deviation from Benford's law, and reported net income losses, for companies large enough (with a revenue of at least 1 billion kuna). Finally, we suggest application of the methodology to improve the internal processes, efficiency and effectiveness of the State Auditing Office.

Data availability: The data used in the study are corporate data in the public domain. For legal reasons, however, the identities of the companies are disguised. Contact the first author for the sanitized data sets that can be used to verify and replicate the analysis.

Keywords: Benford's law, public companies, reporting, fraud detection, auditing

1 INTRODUCTION

The usage of statistical methods in the analysis and detection of fraud in financial reporting is becoming widespread and necessary. Perhaps the simplest and best known, but still effective test is based on Benford's law. Newcomb (1881) and later Benford (1938) noted that in a sufficiently large collection of numerical data expressed in a decimal form, the distribution of occurrences of first digits is not uniform (we explain Benford's distribution in more detail in section 2). Many authors, including Carslaw (1988), Guan et al. (2006), Kinnunen and Koskela (2003), Nigrini (2005), Niskanen and Keloharju (2000), Skousen et al. (2004), Thomas (1989) and Van Caneghem (2002, 2004) investigated the applicability of this fact in accounting and auditing, and specifically in the detection of "cosmetic earning management". It has been shown that the distribution of first digits in financial reports normally complies with Benford's law. If, however, there have been a-posteriori "cosmetic interventions", then for both statistical and psychological reasons, the distribution of first digits changes. As a result, the likelihood of "cosmetic earning management" can be statistically verified, and at least theoretically, the reliability of a specific financial report can be quantified.

In Croatia, we applied these statistical fraud detection tools to the financial reports of 7 large Croatian public and state-owned companies, and one publicly traded company with a substantial share owned by the state of Croatia. By analyzing publicly available data for the years 2010 and 2011, we found that at a level of significance 1% and less, the financial reports of 7 (out of 16) annual reports deviate from Benford's law.

While these statistical results are not conclusive evidence of accounting and reporting fraud, they should be used as an indication for further focused investigation of the State Auditing Office (SAO).

We explain the methodology of application of Benford's law and its limitations in section 2, including the discussion on why Benford's law is expected to appear in annual reports for companies large enough. We also put our research in the context of similar analysis done, e.g. by Nigrini and Mittermaier (1997) for accounting data, and by Rauch et al. (2011) in analyzing reported Greek macroeconomic data. We then present our findings in section 3.

In section 4, we consider whether our suggested tests and other more sophisticated statistical tools could be used by the State Auditing Office when planning audits, managing internal resources, and performing actual audits. We also propose some changes to the processes and applicable laws regarding SAO in section 4.

2 THE APPROACH, METHODOLOGY AND EXAMPLES

2.1 BENFORD'S DISTRIBUTION IN FINANCIAL REPORTS

Perhaps counter-intuitively, the frequency of occurrence of first digits in a random collection of data is very often not uniform. It was noted first by Newcomb (1881) that "how much faster the first pages [of logarithmic tables] wear out than the last ones", with a heuristic explanation in the form of Benford's law. Physicist Frank Benford (1938) rediscovered the law. Benford showed that in 20 different tables, "including[1] such diverse data as areas of 335 rivers, specific heats of 1,389 compounds, American League baseball statistics and numbers gleaned from Reader's Digest articles", the occurrence of first digits obeys Benford's law. Several authors, most rigorously Hill (1995a, 1995b) with an explicit statistical derivation, demonstrated that in a collection of approximately independent data of different orders of magnitude, the frequency of the first digit d=1,…,9 is approximately

$$\pi(d) = log_{10}\left(\frac{d+1}{d}\right) \tag{1}$$

(see table 1 for the actual values).

We can thus observe Benford's distribution of first digits in many statistical samples, e.g. the heights of the tallest buildings in the world; the production of copper/country, and so on. Benford's distribution does not occur if the observed values are from a relatively small range, or if the numbers are assigned (e.g. telephone numbers) or fabricated by people, cf. Nigrini (2000). The theoretical reason for this is that the Benford distribution is the only distribution of first digits (see Benford, 1938) invariant for scaling; that is, the distribution does not change if we for example change the currency. This was rigorously shown by Pinkham (1961).

[1] Summary from Hill, 1995b.

Hill relatively recently gave a more precise theoretical foundation for the ubiquity of Benford's law (Hill, 1995b). He proved that the sum of independent random variables, which themselves have different (random) distributions, has Benford's distribution for sufficiently large samples and collections of random variables. Mathematically, the distribution of the sum of random variables converges in distribution to Benford's distribution, which is a form of the Central Limit Theorem (Hill, 1995b:360)[2]. Hill then concludes (1995b:354) that "This helps explain why the significant-digit phenomenon appears in many empirical contexts, and helps explain its recent application to computer design, mathematical modeling, and detection of fraud in accounting data."

The methods of application of Benford's law in auditing and tax auditing have been developed by Möller (2009), Nigrini (1996), Nigrini and Mittermaier (1997) and Watrin et al. (2008). As a result, today these statistical methods are actively used for example by the Internal Revenue Service in the USA, by the "big four" international auditing companies, and have been implemented as a standard tool in market-leading auditing software tools (e.g. "ACL data analytics"). While these statistical methods cannot prove fraud in financial reporting conclusively, as it is not *a-priori* clear (though probable as explained below) that a financial report obeys Benford's distribution, it can be at the very least used as a "warning tool". Tax offices and auditing companies use it to identify areas where detailed analysis is required, and so manage its resources and accuracy with much more efficiency.

For companies large enough, financial reports are typically assembled as a collection of individual financial reports of different company units. These units are typically of different sizes, have different business scopes, thus the individual distributions of first digits in financials by business unit is expected to be different. By the Hill criterion (1995b), the cumulative financial reports for the entire company should then comply with Benford's law if the company is large and complex enough.

Table 1
Benford distribution of first digit, $\pi(d)=log10((d+1)/d)$

D	1	2	3	4	5	6	7	8	9
π(d)	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046

The statistical tests developed to analyze Benford's law typically start with testing the following standard criteria for the applicability of Benford's distribution (Nigrini and Mittermaier, 1997):

– median is smaller than the arithmetic mean,
– skewness is positive.

[2] Hill, 1995b, p. 360: "Roughly speaking, this law says that if probability distributions are selected at random and random samples are then taken from each of these distributions in any way so that the overall process is scale (or base) neutral, then the significant-digit frequencies of the combined sample will converge to the logarithmic distribution."

Following that, various statistical tests measure the discrepancy of data sets from Benford's law. Given a particular sample, such as a collection of financial numbers from a financial report (balance sheets, profit and loss accounts, cash flows, notes to the report), the appropriate statistical test for the Benford distribution fit is the Pearson χ^2-statistics with 8 degrees of freedom (Ramachadran and Tsokos, 2009, section 7.6.3).

$$\chi^2 = n\sum_{i=1}^{9}\frac{(\pi(d)-r(d))^2}{\pi(d)} \tag{2}$$

here n is the size of the sample, $\pi(d)$ is the Benford probability of occurrence of the digit d, and $r(d)$ is the actual relative frequency of occurrence of the digit d in the data sample. The χ^2 statistics is then used to verify the hypothesis of Benford data fit and report reliability. Thus the null hypothesis is rejected at 5% significance if the χ^2 statistic exceeds 15.51, 1% significance, if χ^2 exceeds 20.09, and at 0.1% significance, if the χ^2 exceeds 26.13. An illustrative but not entirely accurate interpretation is that it represents 95%, 99%, respectively 99.9% likelihood of fraud.

Another statistics, standardly used for verification of whether the frequency of certain digit significantly varies from Benford's law is Z statistics:

$$Z_d = \sqrt{n}\,\frac{|\pi(d)-r(d)|-1/(2n)}{\sqrt{\pi(d)(1-\pi(d)}}, \tag{3}$$

where d=1,…,9 is a fixed digit, and n, $\pi(d)$, $r(d)$ is as above. This is for example used in Nigrini and Mittermaier (1997), and further explained in Durtschi, Hilison and Pacini (2004). As checking and analyzing the values of Z_d statistics for each value of d is beyond the scope of this communication (this is, for example, typically used to pinpoint locations of possible fraud more precisely), we consider the average of Z statistics for all digits

$$Z = \frac{1}{9}\sum_{d=1}^{9} Z_d. \tag{4}$$

Here the hypothesis rejection values are as they usually are for Z-statistics.

It is noted for example by Rauch et al. (2011) that χ^2 statistics is also typically larger for larger samples. To compensate for this and verify the methodology, we also calculate χ^2/n, where n is the size of the sample for each country. In addition, we consider an alternative statistics, the normalized Euclidian distance measure

$$d^* = \frac{\sqrt{\sum_{d=1}^{9}((\pi(d)-r(d))^2}}{\sqrt{\sum_{d=1}^{8}\pi(d)^2+(1-\pi(9))^2}} \tag{5}$$

as in Cho and Gaines (2007) and the distance measure

$$a^* = \frac{|\mu_e - \mu_b|}{9 - \mu_b} \tag{6}$$

where μ_e and μ_b are the average of the first digit in the data and for Benford's distribution respectively, as in Judge and Schechter (2009).

Several authors (e.g. Nigrini and Mittermaier, 1997) also use analysis of frequencies of second, third digits, etc. The Benford distribution of second digits is also non-uniform, and is expected to appear in data samples by the same argument as in Pinkham (1961) and Hill (1996). (One of the authors in Slijepčević (1998) explicitly calculated second digit Benford's frequencies, and proved that they appear in certain series of numbers.) However the variation in Benford's frequencies of second digits is much smaller than the variation of first digits (and this difference diminishes with the third digit and so on), and typically requires large data samples (over 10,000 data points in individual samples in e.g. Nigrini and Mittermaier, 1997).

Finally, we note the similarity of our techniques to those of Rauch et al. (2011), who studied macroeconomic data for various European Union (EU) companies. In a way similar to ours, Rauch et al. (2011) studied a limited set of publicly available (mostly) financial data of different meanings and types, and obtained results quantitatively close to ours. This is similar to our approach to the outside-in analysis of published financial reports, and differs from e.g. Nigrini and Mittermaier (1997), a work analyzing confidentially obtained data of the same type.

2.2 THE METHODOLOGY

We focus in our analysis on publicly available data, obtained from annual reports of large companies. Our approach, partially by necessity-driven by the availability of data, differs somewhat from the typical approach in the literature when accounting data are statistically checked for fraud. Typically authors analyze large data sets of transactions of the same or similar type obtained from the companies themselves and not publicly available. For example Nigrini (1996) analyzes the amounts of total interest paid from 200,000 tax returns; and Nigrini and Mittermaier (1997) consider the sample of over 30,000 invoices paid by the same company.

We, however, choose to use all financial data[3] from publicly available financial reports. We then test the hypothesis H_0: the distribution of first digits in financial reports obeys Benford's law.

[3] In practice, we data mined all the numbers from financial reports (available in pdf format), and then manually excluded all non-financial data (such as years, numbers of employees, etc.).

We explained in the previous section why financial data of large companies should obey Benford's distribution. A valid question is whether the companies in our sample are large enough for a meaningful statistical analysis. We argue that this is indeed the case. In section 4 we show that we obtain statistically similar discrepancies from Benford's law for 4 smaller companies in our sample (defined in accordance to revenue, as these with annual revenue less than 1 billion kuna), and for 4 larger companies. If the studied companies had been too small for this analysis, the statistical deviation would have been significantly higher for smaller companies, as we would not have reached the threshold for which the Hill argument (1995b) applies.

Our approach consisted of five steps: (1) data mining, (2) descriptive statistics and verification of Benford's law in the entire sample, (3) application of χ^2 statistics, Z statistics and other tests to individual financial reports, (4) ranking of the companies according to χ^2 and χ^2/n statistics, and finally (5) consideration of correlations of findings with the company size, the amount of data, and reported financial results.

Our intention was to analyze all available data for Croatian public companies and institutions, with at least several hundred (in our case at least 300) financial data points. Only 7 public companies satisfied that criteria. We also added one publicly traded company with a large share owned by the State of Croatia (the State is the controlling shareholder), to the sample.

For legal reasons, we do not reveal the names of the companies. In this analysis we denote them consistently with letters A, B, C, D, E, F, G, H. We sorted them so that A is the largest company by annual revenue, and H the smallest. Here A, B, C, D are the "larger" companies, with annual revenues exceeding 1 billion kuna, and E, F, G, H are "smaller" companies, with annual revenues less than 1 billion kuna; but in no case less than 300 million kuna. The companies in the sample cover a range of industries, including financial services, energy, transport and infrastructure, and consumer goods.

Unfortunately, available data for some major Croatian public companies, including Hrvatske šume (National Forests), Hrvatske vode (National Water Company), Hrvatska lutrija (National Lottery), and HAC (Croatian Motorways), were not sufficient, as the published annual reports typically include only sparse balance sheet and profit and loss summaries without details. The published reports on audits of various ministries, public institutions and companies on the web page of the State Auditing Office (www.revizija.hr) contained in all instances we looked into also too few data points. We would be happy to extend our analysis if given access to further information.

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The first step, data mining, included developing a simple software tool to extract all data from an annual report in a PDF format, downloaded from web pages of respective companies. All the data had to be manually checked for consistency. We had to manually verify that the extracted data contain only financial information, and exclude numbers such as years, percentages, etc. We then applied several tools of descriptive statistics as a first check of applicability of Benford's law. We first noted that the cumulative data (n=24,596) from the financial reports for all eight companies for years 2010 and 2011 indeed fit Benford's distribution as expected. We also demonstrate that all data sets pass the median vs. arithmetic mean, and skewness tests.

We then ranked the companies and reports (all 16 data samples), in accordance to all five statistics χ^2, Z, χ^2/n, d^* and a^*. Our analysis will show that rankings are consistent.

We also considered testing the frequency of second (and later) digits. However as explained in section 2.1, our data samples are too small to statistically significantly detect variations in these distributions.

We finally consider as the final step whether our analysis can be interpreted in such a way as to indicate likelihood of fraud. More precisely, our hypothesis is H_0: discrepancy from Benford's law is not correlated with the reported net income; while its alternative is H_1: discrepancy from Benford's law is positively correlated with reported losses. The rationale for this is that both the accounting fraud, and reported losses of public companies are expected to be correlated with less competent management and corruption. In other words, we conjecture that public companies reporting losses would report even worse financial results without "cosmetic management" of reports.

We analyze this by first considering the correlation of deviation from Benford's law, measured by χ^2 and Z statistics, with the company size. Then we consider the correlation of likelihood of fraud and reported net losses. As our data set is limited and the actual net income numbers are disguised in this analysis, we study correlation by comparing rankings rather than more precise tools such as regression.

3 FINDINGS: STATISTICAL ANALYSIS OF ANNUAL REPORTS

3.1 APPLICABILITY OF THE METHODOLOGY

Prior to applying the statistical tests to the analyzed financial data of eight large public companies in Croatia, we confirmed that Benford's law statistical tests are probably applicable. Firstly, each data set contained at least 300 individual numbers (see N in table 4). Here we included only financial data, and excluded as

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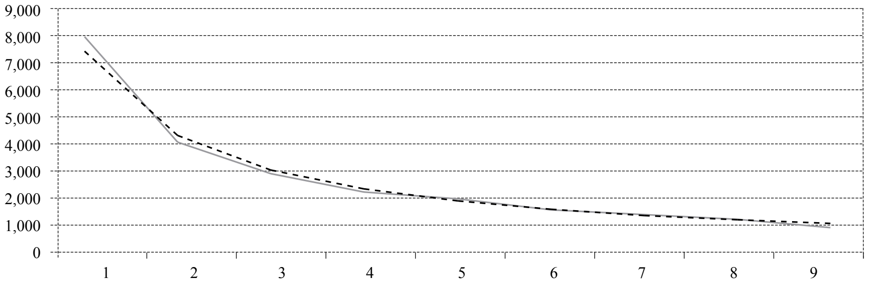
required all other numbers from the financial report. In each instance, we verified that the financial data in the sample indeed includes the values of several (at least four) orders of magnitude.

Figure 1

First digit frequency in the cumulative sample (below)



Note: We plot the frequency of first digits in the cumulative data set (n=24,596) – full line, compared with the expected Benford's law frequency – dashed line.

We then showed that the aggregate data set, that means all the financial data from all 16 financial reports, complies fairly well with Benford's law, as shown in figure 1 and table 2.

Table 2

First digit frequencies in the aggregate data sample (observed and predicted by Benford's law)

d	Observed	Benford
1	7,911	7,400
2	4,084	4,328
3	2,944	3,071
4	2,280	2,382
5	2,019	1,946
6	1,627	1,646
7	1,459	1,426
8	1,275	1,257
9	982	1,125
Total	24,581	24,581

We compared for each of 16 data sets whether the arithmetic mean is larger than median, and whether skewness is positive, as these are standard "tell-tale" signs for Benford's law (Nigrini and Mittermaier, 1997). As shown in table 3, each of 16 data sets satisfies these criteria. In table 3 we also for illustrative purposes show these numbers for the aggregate data set, and for the actual theoretical Benford distribution. In addition, we list ratios of frequencies of digits 1 versus 2, and 1 versus 9, confirming Benford-like behavior.

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TABLE 3
Arithmetic mean, median, skewness, ratios of frequencies if 1 vs. 2, 1 vs. 9; all for 16 data samples, the aggregate data sample, and the Benford distribution

Company	Year	Mean	Median	Skewness	1 vs. 2	1 vs. 9
A	2010	3.20	2.0	0.91	2.3	11.2
A	2011	3.38	3.0	0.78	2.1	9.6
B	2010	3.30	3.0	0.90	1.7	7.6
B	2011	3.27	2.0	0.86	1.9	8.9
C	2010	3.39	3.0	0.80	1.9	7.4
C	2011	3.47	3.0	0.74	1.8	7.6
D	2010	4.13	3.0	0.41	2.0	2.5
D	2011	3.30	3.0	0.80	2.8	26.3
E	2010	3.72	3.0	0.56	2.0	7.0
E	2011	3.78	3.0	0.58	1.9	4.6
F	2010	3.41	3.0	0.77	2.0	8.4
F	2011	3.57	3.0	0.70	2.0	5.9
G	2010	3.77	3.0	0.63	1.1	3.8
G	2011	3.40	3.0	0.80	1.5	7.7
H	2010	3.02	2.0	1.04	2.0	13.3
H	2011	3.10	2.0	0.95	2.2	13.1
Aggregate		3.38	3.0	0.80	1.9	8.1
Benford		3.44	3.0	0.80	1.7	6.6

3.2 STATISTICAL RESULTS AND RANKING

As we show in table 4, we found out by applying the χ^2-test, that the annual reports for four companies deviate at the significance level of 0.1% from Benford's law. The same 7 data sets deviate from Benford's law by the average Z statistics at the average significance level of 5% (significance of selected individual digits is thus smaller). We see that in our sample, the χ^2-test and the average Z-test "flag" the same data sets, and also have completely consistent rankings.

An additional argument in favor of the possible unreliability of some financial reports is the relative consistency in the ranking of the 16 analyzed financial reports in accordance with all five statistics considered (χ^2, Z, χ^2/n, d^* and a^*). Table 4 shows that the annual reports of H and D relatively consistently lead the ranking for all the considered statistics. We conclude that the financial reports of H, D, and F for both years 2010 and 2011, and A for the year 2010 should be scrutinized by the authorities.

In table 5 we compare the size of data set and the values of χ^2 and χ^2/n statistics with the aggregate values from Rauch et al. (2011) and find out that our data behave similarly, as the key values are within the factor 2 of the ones reported in Rauch et al. (2011), as shown in table 5. This is relevant, as Rauch et al. (2011) report on data with known "cosmetic management" of reports, and as such a useful benchmark.

Table 4
Results of statistical tests and ranking

						Ranking				
Company	**Year**	**N**	**χ^2**	**Z**	**chi/n**	**χ^2**	**Z**	**chi/n**	**d**	**a***
H*	2010	2,120	111.7	3.02	0.053	1	1	4	5	3
H*	2011	2,230	84.8	2.54	0.038	2	2	5	6	4
D*	2011	294	67.3	2.45	0.229	3	3	1	2	10
D*	2010	293	59.0	1.97	0.201	4	4	2	3	2
F*	2010	2,353	37.0	1.88	0.016	5	5	10	11	15
A*	2010	1,135	32.2	1.70	0.028	6	6	7	7	8
F*	2011	2,577	29.8	1.65	0.012	7	7	12	1	1
E	2010	641	23.3	1.23	0.036	8	8	6	8	7
A	2011	1,272	19.9	1.24	0.016	9	9	11	12	12
G	2010	275	18.6	1.25	0.068	10	10	3	4	6
E	2011	751	16.9	1.17	0.022	11	11	8	9	5
C	2011	4,216	16.1	1.02	0.004	12	12	15	16	16
B	2011	891	14.7	1.03	0.016	13	13	9	10	9
C	2010	3,709	10.9	1.02	0.003	14	14	16	15	13
G	2011	1,027	7.9	0.80	0.008	15	15	14	14	14
B	2010	812	7.7	0.80	0.009	16	16	13	13	11

Note: N – the size of the sample. Statistical tests and their ranking as explained in section 2.2.

** Companies and reports deviating from Benford's law at significance level of 0.1% (χ^2 test), respectively 5% (the average Z test).*

Table 5
Comparison of the aggregate data sets in our sample and Rauch et al.

Aggregate data set	**Size of the sample**	**χ^2**	**χ^2/n**
Our analysis	24,596	80.87	0.0033
Rauch et al. (2011)	39,691	69.64	0.0018

3.3 CORRELATIONS OF RESULTS

As discussed in section 2.1, it is a valid question whether the analyzed companies and data samples are large enough for them to be expected to comply with the Benford distribution. Extrapolating from our limited data sample, it seems that in Croatia, companies with at least 1 billion kuna annual turnover are large enough for meaningful analysis. In table 6, we show the number of data samples identified as significantly deviating from the Benford law, as explained in the previous section (7 in total). We sort them by company size. We see that, at least in our small data sample, there seems to be no significant correlation of company size and compliance with Benford's law. We conclude that at least the four larger companies in our sample are large enough for this type of scrutiny. If this were not the case and the companies in our sample were not large and complex enough for Benford's law to occur (in accordance with Hill's argument discussed in section 2.1), we would have observed a significantly better Benford fit for larger companies. We note that these conclusions are preliminary, and we plan to study them further on larger company samples.

TABLE 6

Number of data sets/reports, by company size

	Small companies**	Large companies**
Not Benford*	4	3
Benford*	4	5

** Not Benford: seven reports in the table 4, significantly deviating from Benford's distribution.*

*** Small companies: up to 1 billion kuna annual turnover. Large companies: over 1 billion kuna annual turnover.*

Finally, we consider whether there is a correlation between the deviation from Benford's distribution and reported net losses. As already explained in section 2, such correlation would indicate that our method of fraud detection is effective. We focus on 4 larger companies, named A, B, C, D, as the previous discussion suggests they should be large enough to comply with Benford's law. In table 7 we list rankings of all eight reports for these companies, in terms of χ^2-test of compliance with Benford's distribution (1 being the most deviant one). We compare this with the ranking of their (pretax) net income/annual revenue ratio (I/R), where 1 correspond to the company with the largest reported losses. Here companies A and D reported losses in both years 2010, 2011; while companies B and C reported positive results. Finally, we rank it in accordance with the reported pretax net income.

TABLE 7

Rankings of companies A-D with respect to χ^2-test compliance with Benford's distribution, pretax net income/annual revenue (I/R), and the pretax net income (I)

Company	Year	Ranking		
		χ^2	I/R	I
D	2011	1	1	1
D	2010	2	2	4
A	2010	3	3	2
A	2011	4	4	3
C	2011	5	8	7
C	2010	6	5	5
B	2011	7	6	6
B	2010	8	7	8

Note: For χ^2-test, companies ranked from the most deviant from Benford's law (1) to the least (8). For N/I, I, the companies ranked from these with the largest reported losses (1) to the ones with the most positive result (8).

We find it significant and revealing, that indeed the rankings in table 7 seem to be correlated. In particular, the 4 reports with the largest deviation from Benford's law are also the 4 reports with annual net losses, and their rankings of χ^2-test (and average Z-test) and I/R coincide. A possible interpretation is that the losses of companies A and D were even larger than reported, and then cosmetically reduced in the final published reports. Again, this analysis is preliminary, and we hope to study it further and confirm for sets of more companies, large enough for a more precise analysis (e.g. linear regression, etc.).

4 SUMMARY AND CONCLUSION

Accounting data are collected from a wide variety of sources. Benford's law, which defines the empirical distribution of first digits in diverse data sets, can be used to detect manipulated data in financial reporting. We showed that the financial data of large Croatian public and state-owned companies do on average comply with Benford's law, while we found indication of reporting manipulation at the significance level of 1% and less for several of them. Furthermore, we found indication of correlation of deviation from Benford's law, and reported losses, which is a further indication of reporting fraud.

Here we outline possible implications and recommendations regarding the operations of the State Auditing Office in Croatia. In table 8 we list the year for which SAO completed the audit for the considered companies. It is perhaps striking that for the eight analyzed companies, we found in total only 5 SAO audits for the last five full reporting and audited years 2006-2011. The only company from the considered ones audited for 2011 is company D, where SAO had a qualified opinion.

TABLE 8
The last years audited by the State Auditing Office

Company	Year
A	2009
B	N/A
C	N/A
D	2011
E	2009
F	N/A
G	2006
H	2007

Source: Državni ured za reviziju.

Independent auditors, however, audit yearly financial statements of the analyzed companies. The auditors auditing analyzed companies include Ernst&Young, BDO Croatia and Deloitte. We found audited annual reports for 7 companies from our sample; and for one of them we could not find a report.

We believe there is a need for more frequent and thorough audits by SAO. We give a few supporting facts. SAO audited in 2012 (i.e. for the year 2011) 23 state owned companies, and issued only 2 unqualified opinions, 20 qualified opinions, and one adverse opinion, as reported in the State Auditing Office (Državni ured za reviziju, 2012). In the specific comparable case of company D, its auditor issued in the annual report an unqualified opinion for 2011, contrary to the SAO opinion. The company management chooses the independent auditor, thus possibly it may have some influence over its opinion in particular in the current challenging market conditions in Croatia. In any case the scope of the SAO audit is typically wider than that of the independent auditors, as it includes for example verification of compliance with public procurement procedures.

SAO audits only the most important and largest public institutions annually and all the others much less frequently, as noted in table 8 and clearly visible in the list of executed audits on www.revizija.hr.

We believe that incorporation of the statistical methods including but not restricted to Benford's law χ^2 and Z statistics could improve effectiveness and efficiency of SAO. For example, SAO could:

- more frequently (at least once annually), in a relatively automated way, look for statistical indications of "cosmetic manipulations" for all public institutions and companies,
- focused audits could then be performed for the subjects with significant deviations,
- statistical methods could also assist auditors in focusing their work when auditing a specific subject.

We believe in such a way SAO could, even with the existing, surely limited and constraining, resources, perform better its public service.

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